Name and Surname : ……………………………………………………………………………..

Grade/Class : 11/……….   **Mathematics Teacher** : …………………………..

Hudson Park High School

GRADE 11

MATHEMATICS

June Examination

Marks : 150		Date : 17 May 2024	
Time : 3 hours			
Examiner : PHL and SLT		Moderator(s) : SLT PHL VNT GRF	

INSTRUCTIONS

1. Illegible work, in the opinion of the marker, will earn zero marks.

2. Number your answers clearly and accurately, exactly as they appear on the question paper.

3. **A blank space of at least two lines should be left after each answer.**

4. An Answer Booklet is provided.

   Fill in the details requested on the front of this Question Paper and the Answer Booklet, before you start answering any questions.

   Hand in your submission in the following manner :

   (on top) **Answer Booklet**
   (below) **Question Paper**

   Please **DO NOT STAPLE** your Answer Booklet and Question Paper together.

5. Employ relevant formulae and show all working out.
   Answers alone *may* not be awarded full marks.

6. (Non-programmable and non-graphical) Calculators may be used, unless their usage is specifically prohibited.

7. Answers must be written in blue or black ink, as distinctly as possible, on both sides of the page. An HB pencil (but not lighter eg. 2H) may be used for diagrams.

8. Round off answers to 2 decimal places, where necessary, unless instructed otherwise.

9. If (Euclidean) GEOMETRIC statements are made, REASONS must be stated appropriately.

PHASE 1

75 marks

QUESTION 1

1.1. Solve for x :

1.1.1. $x(6-x)=0$ (2)

1.1.2. $3x^2-2x-6=0$ (correct to TWO decimal places) (3)

1.1.3. $(3-x)(7+x)<0$ (3)

1.1.4. $\sqrt[3]{32}=8^{3x}.2^{6x}$ (3)

1.1.5. $x-4-2\sqrt{x-1}=0$ (5)

1.1.6. $3x^{\frac{2}{5}}-8=0$ (3)

1.2.1. Show that : $4.3^{1-x}+3^{2-x}=\frac{21}{3^x}$ (3)

1.2.2. Hence, or otherwise, solve for x if : $4.3^{1-x}+3^{2-x}=63$ (3)

1.3. Solve for x and y :

$x^2+2yx=3y^2$ and $2y-x=6$ (6)

1.4. Simplify fully without the use of a calculator :

$\sqrt[p]{\frac{10^p+2^{p+2}}{5^{2p}+4.5^p}}$ (4)

[35]

QUESTION 2

CALCULATORS MAY NOT BE USED IN THIS QUESTION

2.1. Simplify fully : $\sqrt{3}\left(\sqrt{12} - \sqrt{1\frac{1}{3}}\right)$ (3)

2.2. If : $\frac{6}{\sqrt{3}+3} = a + b\sqrt{3}$, calculate the values of a and b. (4)

2.3. If : $3^x = 5,\ 5^y = 7$ and $7^z = 9$, calculate the value of xyz. (3)

[10]

QUESTION 3

3.1. Write down a quadratic equation, in the form $ax^2 + bx + c = 0$ (where $a, b, c \in \mathbb{Z}$), whose roots are -3 and $\frac{5}{4}$. (2)

3.2. The roots of the equation $x^2 - 5x + c = 0$ are given by $x = \frac{5 \pm \sqrt{41}}{2}$. Calculate the value of c. (4)

3.3. Discuss the nature of the roots of the equation :

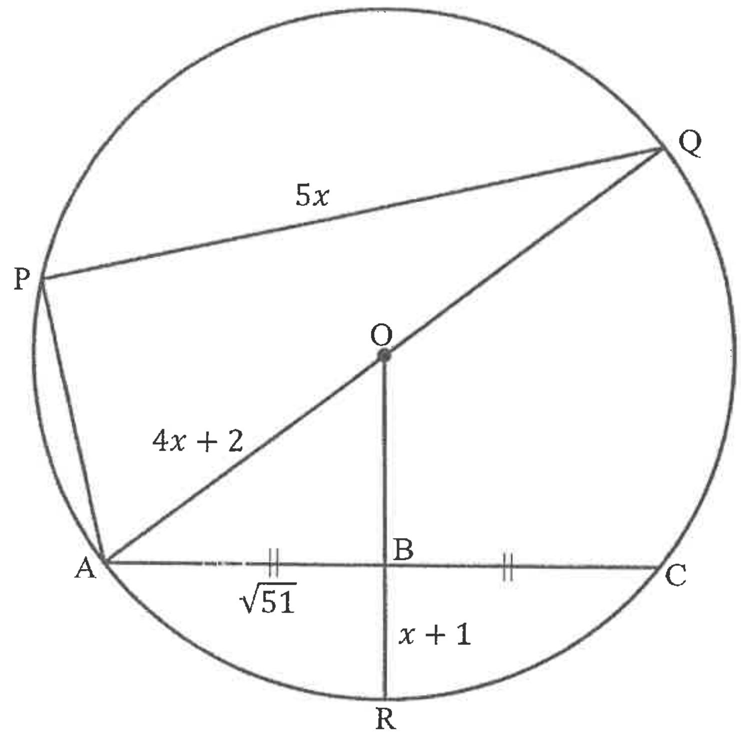
$$x^2 + kx = k + 2x$$

where $k \in \mathbb{R}$. (4)

[10]

QUESTION 4

4. O is the centre of the circle. AB = BC = $\sqrt{51}$. AO = $4x + 2$, BR = $x + 1$ and PQ = $5x$:



Calculate the :

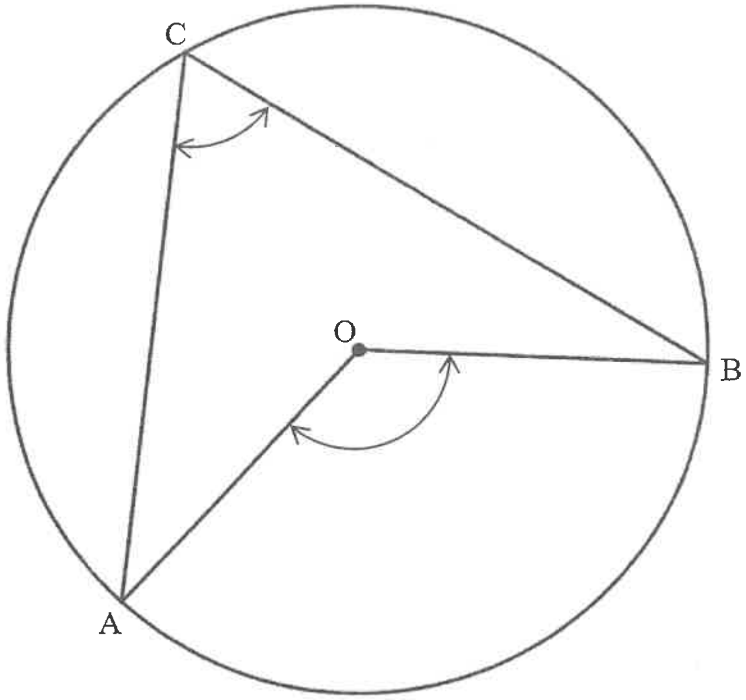
4.1. value of x, showing clearly that it will be 2, and (5)

4.2. hence, length of PA. (3)

[8]

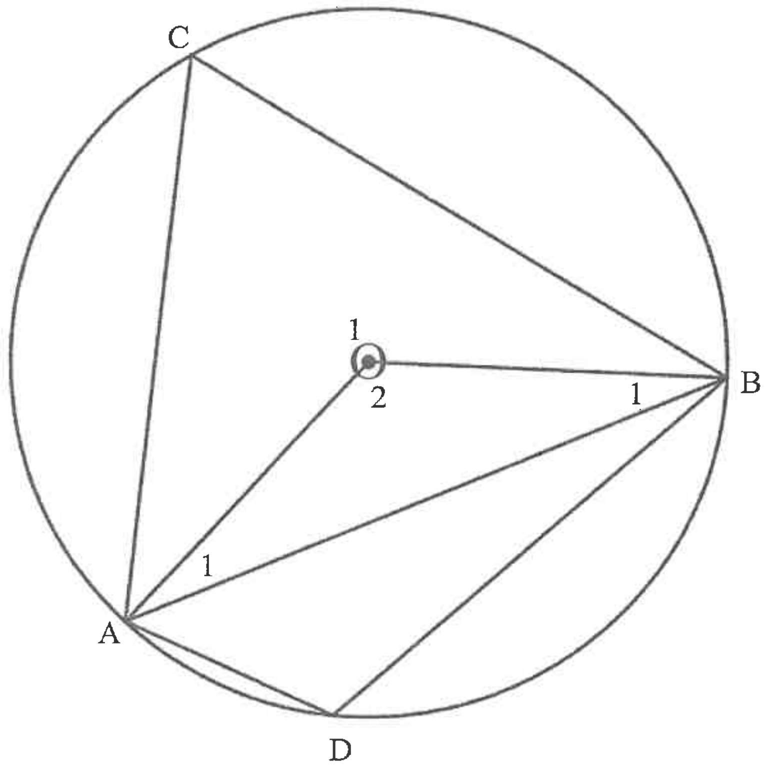
QUESTION 5

5.1. O is the centre of the circle.



Prove the THEOREM which states that : $A\hat{O}B = 2.A\hat{C}B$. (5)

5.2. O is the centre of the circle. $\widehat{O}_1 = 330° - 3x$ and $\widehat{C} = 2x - 5°$:



Calculate the :

5.2.1. value of x, and (4)

5.2.2. hence, size of $\widehat{B}_1$. (3)

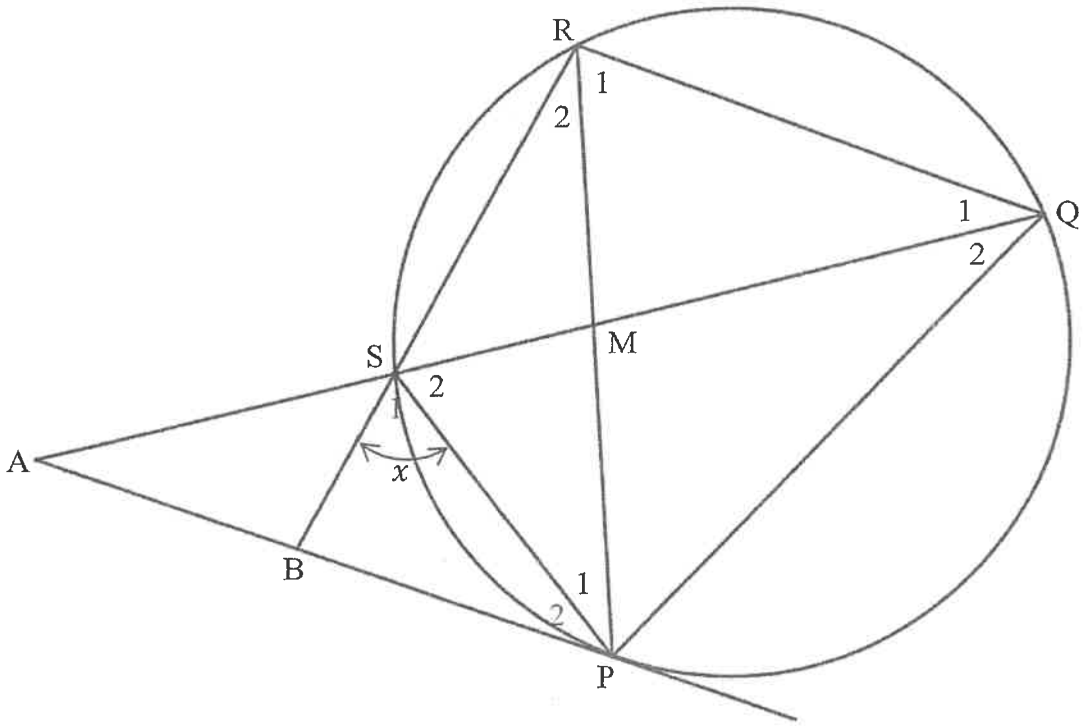
[12]

PHASE 2

75 marks

QUESTION 6

6. AP is a tangent to the circle. PR = PQ. Let $\hat{S}_1 = x$:



Prove that :

6.1. $\hat{S}_1 = \hat{S}_2$ (5)

6.2. MP is a tangent to the circle, passing through points P, S and A, at P. (6)

[11]

QUESTION 7

CALCULATORS MAY NOT BE USED IN THIS QUESTION

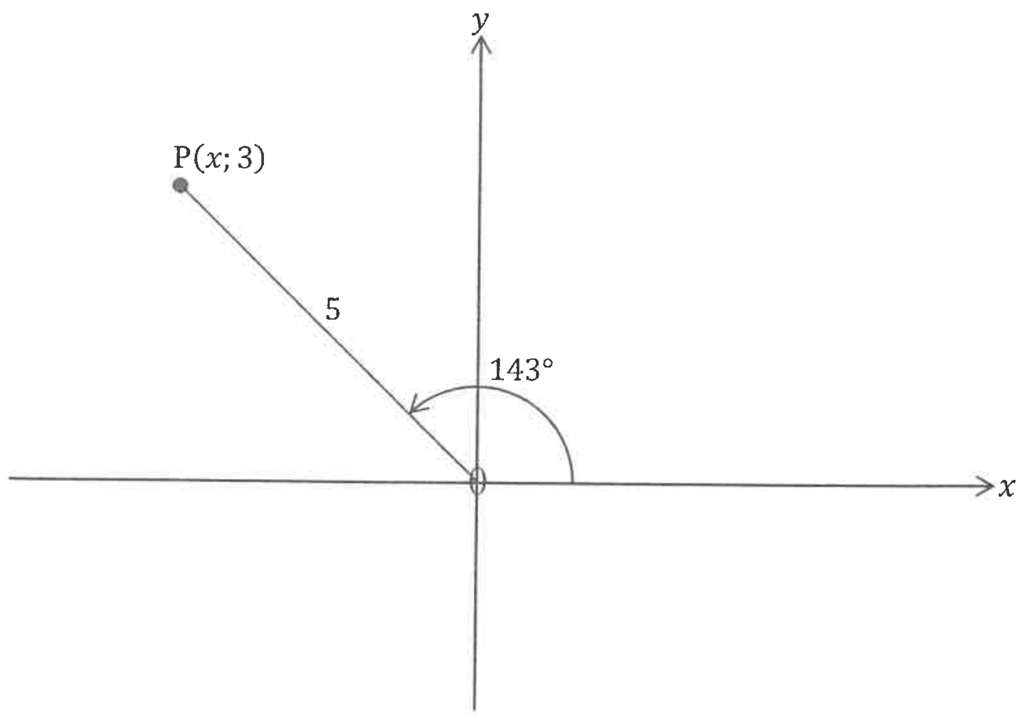
7.1. Given : $\sin 25° = k$ where $0 < k < 1$.

Determine the following in terms of k :

7.1.1. $\tan 25°$ (2)

7.1.2. $\cos(-25°)$ (2)

7.2. In the diagram below : $x\hat{0}P = 143°$, $P(x; 3)$ and $0P = 5$.



Calculate the values of :

7.2.1. x (1)

7.2.2. $\sin 143°$ (1)

7.2.3. $\cos 37°$ (2)

7.2.4. $\tan 53°$ (2)

7.3. Given : $\tan 150°$

7.3.1. Draw the special diagram used to evaluate trigonometric ratios of 30°. (1)

7.3.2. Hence, determine the value of : $\tan 150°$. (2)

7.4. Simplify fully, to contain only one trigonometric ratio of x :

$$\frac{\sin^2(x-180°) + \frac{\tan 197°}{\tan 343°}}{1 + \cos(1530°+x)}$$ (6)

7.5. Factorise fully : $7\sin^2 x + 4\sin x\cos x - 4$ (3)

7.6. Given the identity : $\cos x\left(\tan x + \frac{1}{\tan x}\right) = \frac{1}{\sin x}$

7.6.1. Prove the identity. (4)

7.6.2. For which value(s) of x will the identity not be valid ? (4)

[30]

QUESTION 8

8.1. The general solution of a certain trigonometric equation is known to be

$x = 25° + k.70°;\ \ k \in \mathbb{Z}$

Calculate the solutions of the equation in the interval $x \in [-180°; 180°]$. (1)

8.2. Solve for x :

8.2.1. $2\cos x = -\sin 1040°$ (3)

8.2.2. $3\sin 5x + \cos 5x = 0$ (3)

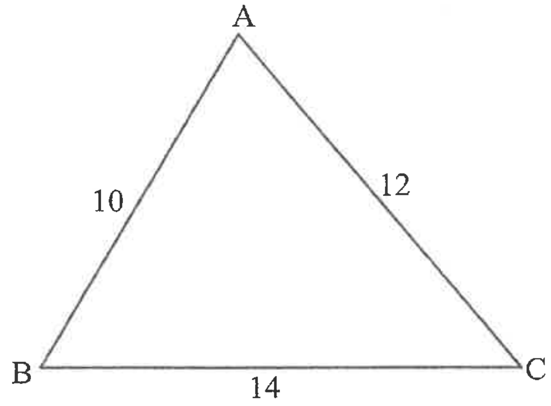
8.2.3. $\sin(x + 10°) + \cos 2(x - 15°) = 0$ (4)

8.2.4. $-3\tan x = 2\cos x$ (8)

[19]

QUESTION 9

9.1. In the given triangle :

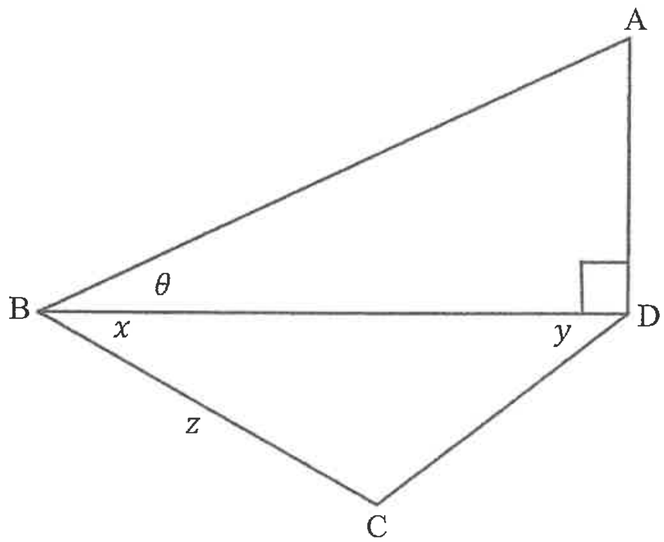


Calculate the :

9.1.1. size of $\hat{B}$ (3)

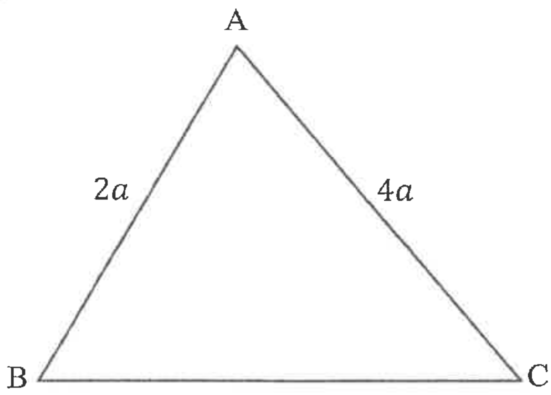
9.1.2. area of ΔABC. (2)

9.2. In the given triangle, AD $\perp$ BD :



Prove that : $AD = \dfrac{z \tan\theta \sin(x+y)}{\sin y}$ (4)

9.3. In the given triangle, $\hat{A} = 180° - x + y$:



9.3.1. Prove that : $BC = 2a\sqrt{5 + 4\cos(x - y)}$ (4)

9.3.2. Hence, calculate BC, if $a = 10$, $x = 50°$ and $y = 12°$. (2)

[15]

5. INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad$ $area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$